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Development of a Progressive Failure Model for Strength of Laminated Composite Structure

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13. ABSTRACT (Maximum 200 words)

This report briefly describes the background information and current work plan for a multiyear effort to develop a progressive failure model for predicting strength of laminated composite structure under static loads and fractured in the in-plane failure modes. It also highlights the intermediate results obtained in FY89. Keywords: Stress analysis, Tensile Strength, Stiffness, Degradation, Mathematical Models, PDHOLE Computer Programs. (AW)

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1.0 INTRODUCTION

With the advances of the composite materials technology, the use of high-performance laminated composites for naval structures has been substantially increased in recent years. For examples, E-glass/epoxy and graphite/epoxy composite laminates, respectively, have been used by Naval Ocean Systems Center to build transducers and advanced unmanned deep-ocean search systems (AUSS). In designing these laminated composite structures, it is vitally important to have a model that can accurately predict the strengths of the structures under service conditions. One such model, the progressive failure model, can take into account the progressive damage and fracture process of composite laminates. The model is usually incorporated into a finite element structural analysis code for composite structures due to the complexities of the structural problems encountered.

An extensive literature review was conducted in FY88 (via an independent research (IR) program) on various progressive failure modeling schemes (references 1 to 7). Correlations of the analytical results predicted by these progressive failure models with the experimental data on composite specimens with complicated geometries (e.g., hole, slit, crack, etc.) and fractured in the in-plane modes (e.g., fiber breakage, matrix cracking, etc.) were found not totally satisfactory. The ply failure criterion used in these models may be the prime reason for this. Meanwhile, an improved multiaxial failure criterion, the piecewise quadratic strength tensor criterion (references 8 and 9), had been recently developed. Clearly, the introduction of the improved ply failure criterion into the progressive failure model should certainly improve the accuracy in predicting the strength of laminated composite structure. In light of this, it was proposed to use the improved multiaxial failure criterion to implement a progressive failure model in a finite element code for predicting the strength of laminated composite structure under static loads and fractured in the in-plane failure modes.

The next section provides the background information for the progressive failure model implementation. Section 3 discusses the implementation approach resulting from FY88's IR effort, formulating the work plan for FY89 and FY90. Section 4 highlights the intermediate results obtained in FY89.

2.0 BACKGROUND

For background information, this section briefly reviews the concepts for composite material stress analysis (references 10, 11, and 12), the piecewise quadratic strength tensor failure criterion (references 8 and 9), and progressive failure model (references 1 to 7). For more detailed account of these concepts, including the notation conventions, one may refer to the references cited above.

2.1 COMPOSITE STRESS ANALYSIS

A lamina (ply), as shown in figure 1(a), is a flat (sometimes curved as in a shell) arrangement of unidirectional fibers or woven fibers in a matrix. A laminate (shown in figure 1(b)) is a stack of laminae with various orientations of fiber directions (called ply orientations) in the laminae.

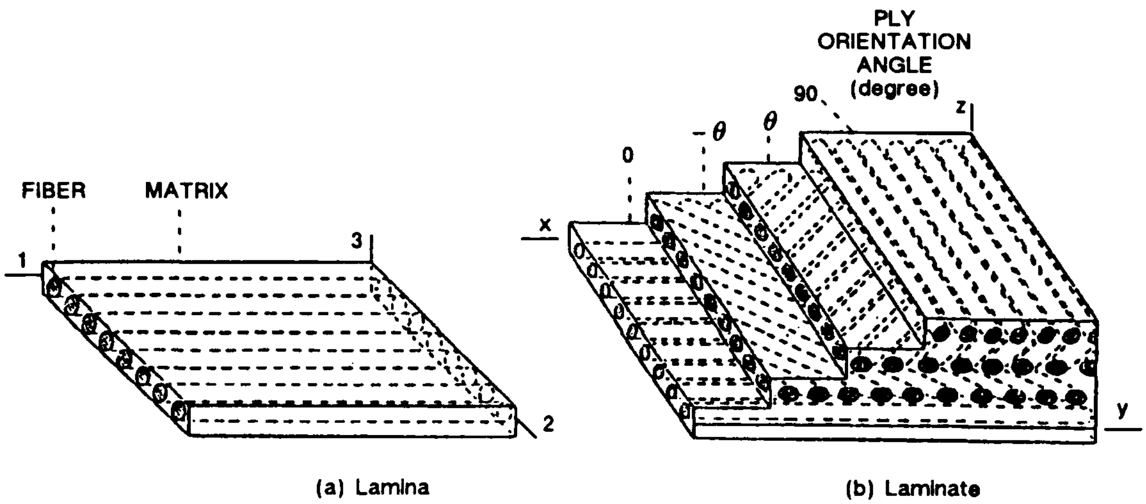
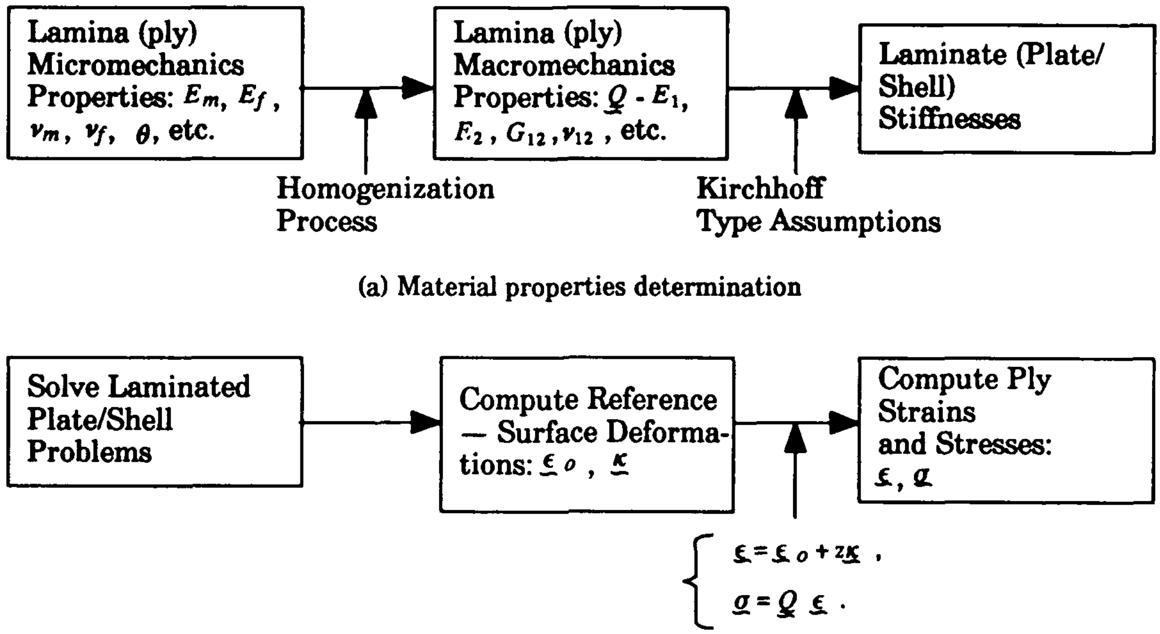


Figure 1. Laminated composite.

Because of the inherent heterogeneous nature of fiber-reinforced composite materials, they are conveniently studied from two points of view: micromechanics and macromechanics. In micromechanics, composite material behavior is studied by examining the interaction of the constituent fibers and matrixes on a microscopic scale. In macromechanics, composite material behavior is studied by presuming the material to be homogeneous and detecting the effects of the constituent materials only as averaged apparent properties of the composite. Use of both the concepts of micromechanics and macromechanics allows the tailoring of a composite material to meet a particular structural requirement with little waste of material capability. The ability to tailor a composite material to its job is one of the biggest advantages that composites have over metallic or plastic structures.

Figure 2 depicts the computational sequences for composite stress analysis, including those for laminated plate/shell material properties determination and ply strains/stresses calculation. First, the micromechanics properties (matrix Young's modulus and Poisson's ratio, E_m and ν_m; fiber Young's modulus and Poisson's ratio,

E_f and ν_f; and ply orientation angle, θ) of each ply in the laminate under consideration are homogenized to obtain the ply apparent or macromechanics properties (ply (reduced) stiffness matrix, $\underline{Q}$, whose components are computed based on the Young's modulus in the fiber direction, E_1; the Young's modulus in the direction transverse to the fibers, E_2; the in-plane shear modulus, G_{12}; the major Poisson's ratio, ν_{12}, etc.). Second, the macromechanics properties of all the plies are integrated through the thickness of the laminate by means of Kirchhoff type of assumptions (e.g., linear strain variation through the thickness) to obtain the laminate (plate/shell) stiffnesses.



(b) Ply strains/stresses calculation

Figure 2. Composite stress analysis.

With the laminate material properties so determined and boundary conditions (loads and/or displacements) applied, the associated laminated plate or shell boundary-valued problem can be solved by the conventional plate/shell theories. The solutions contain reference-surface (usually midsurface of the laminated plate/shell) deformations: strains, $\underline{\epsilon}_o$, and curvatures, $\underline{\kappa}$. Finally, the strains, $\underline{\epsilon}$, and stresses, $\underline{\sigma}$, for each ply can be computed by

$$\underline{\epsilon} = \underline{\epsilon}_o + z\underline{\kappa}\,, \tag{1}$$

$$\underline{\sigma} = \underline{Q}\,\underline{\epsilon}\,, \tag{2}$$

where z is the coordinate along the thickness direction.

2.2 THE PIECEWISE QUADRATIC STRENGTH TENSOR FAILURE CRITERION

For a general anisotropic composite, the piecewise quadratic strength tensor failure criterion can be written as[1]

$$f(\sigma_k) = F_i\sigma_i + F_{ij}\sigma_i\sigma_j + H_i\sigma_i \mid H_j\sigma_j \mid = 1, (i, j, k = 1, \ldots, 6), \tag{3}$$

where f is a scalar function, σ_k is the contracted notation of the second-rank stress tensor,[2] F_i and H_i are strength tensors of rank two, and F_{ij} is a strength tensor of rank four. In addition, constraints (usually referred to as the stability conditions) must be imposed on the strength tensors F_{ij} and H_i to ensure that the material strength is finite in all directions. More specifically, these constraints are

$$(F_{ij} + H_iH_j)\sigma_i\sigma_j > 0 \tag{4}$$

for all stress points σ_i in the half space

$$H_i \sigma_i \geq 0, \tag{5}$$

and

$$(F_{ij} - H_iH_j)\sigma_i\sigma_j > 0 \tag{6}$$

for all stress points σ_i in the other half space

$$H_i \sigma_i < 0 . \tag{7}$$

Geometrically, the failure surface represented by equation (3), with the strength tensors F_{ij} and H_i satisfying the stability conditions given by equations (4) and (6), is a piecewise ellipsoid in the six-dimensional stress space, which consists of two ellipsoids in the two half spaces defined by equations (5) and (7). Hence, the failure criterion represented by equation (3) has been referred to as the piecewise quadratic failure criterion.

The above results hold for a general anisotropic solid. They have been reduced to an orthotropic material, a transversely isotropic material, and an isotropic material for application to composites with various material symmetries. The reduced

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[1] Unless otherwise indicated, the usual summation convention over a repeated index is used throughout this report.

[2] With reference to a rectangular Cartesian coordinate system (i.e., xyz or, equivalently, $x_1x_2x_3$ system): $\sigma_1 = \sigma_x, \sigma_2 = \sigma_y, \sigma_3 = \sigma_z, \sigma_4 = \sigma_{xy}, \sigma_5 = \sigma_{yz}, \sigma_6 = \sigma_{zx}$.

results, including explicit expressions for the strength criteria and the restrictions imposed on the components of the strength tensors occurring in these criteria, can be found in references 8 and 9.

2.3 PROGRESSIVE FAILURE MODEL

Estimates for ultimate strengths of fiber-reinforced laminates based on first ply failure concept are often highly conservative because first ply failure usually does not result in the total failure of the laminate. To make reasonable strength prediction at ultimate failure of the laminate, account has to be taken into the progressive damage of the laminate and the resulting stress redistribution that can occur due to such damage.

To show how these can be taken into account by a progressive failure model, the numerical procedures for the model incorporated in a finite element code are given in figure 3. These procedures are performed according to the following steps:

1. Increase the applied load/displacement by a small increment and perform composite stress analysis as described in subsection 2.1 to obtain strains/stresses for each element in each ply of the laminate.

2. Access damage by comparing the computed stresses/strains with the ply (and/or interply) failure criterion. Return to the first step if no damage is found. Continue to the following steps if damage occurs.

3. Degrade (modify) ply or interply macromechanics properties (e.g., ply stiffness $\underset{\sim}{Q}$) for the damaged (failed) elements.

4. Perform composite stress analysis for stress redistribution.

5. Repeat steps 2 to 4 until no more damages can be found at the given load/displacement increment.

6. Repeat steps 1 to 5 until equilibrium can no longer be achieved or no more load can be sustained.

From figure 3, it should be clear that the following modules are essential for a progressive failure model implemented in a finite element code:

a. Progressive failure modeling driver for load increment loop and stress redistribution loop at a given load increment,

b. Ply failure criterion,

c. Stiffness degradation model,

d. Damage propagation output.

Before leaving this section, note that various schemes of progressive failure modeling differed from one another primarily in the ply failure criterion as well as the degradation model of the stiffness of the laminate under failure of one or more plies.

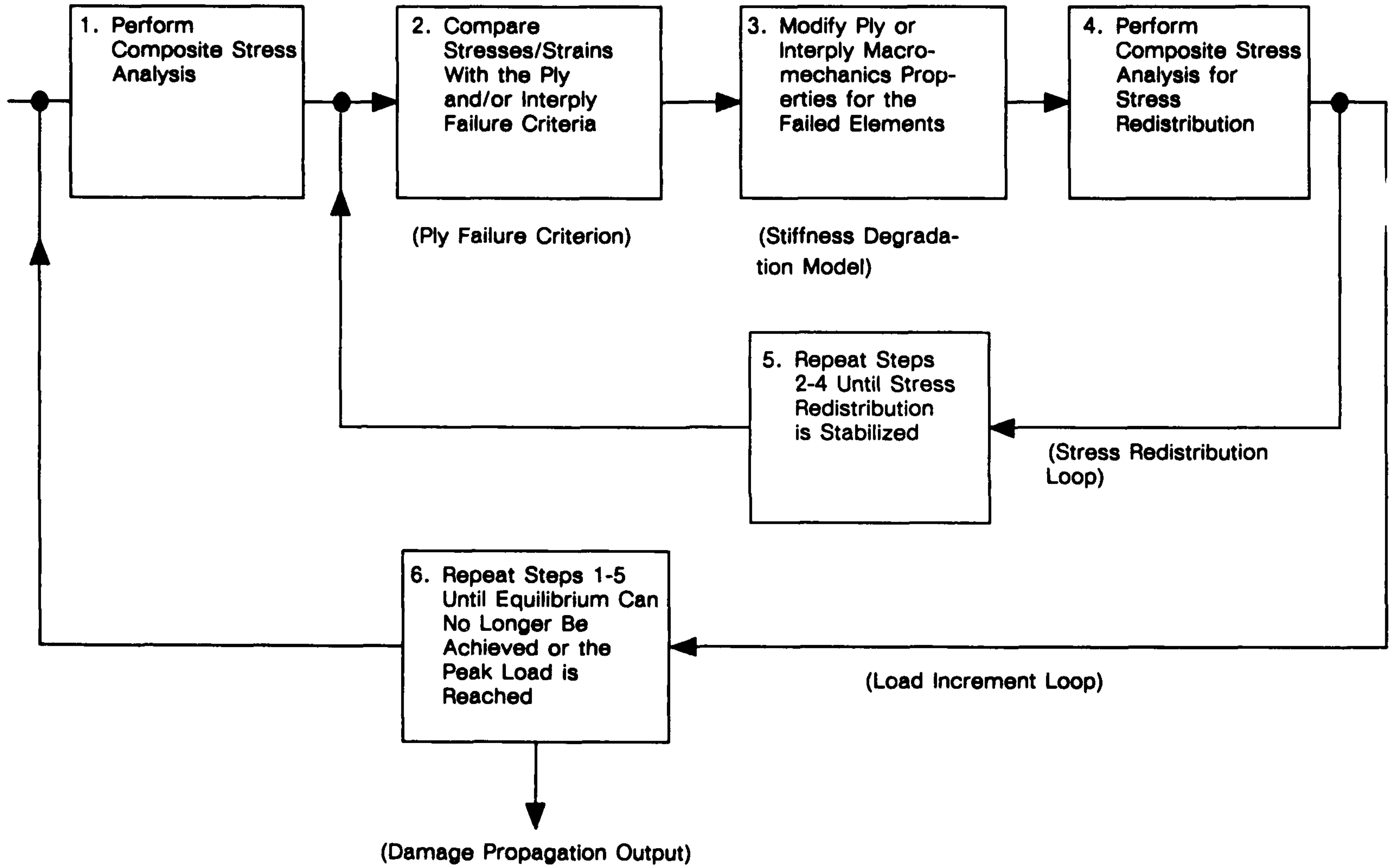

Figure 3. Progressive failure model.

3.0 DEVELOPMENT PLAN

To implement the proposed progressive failure model into a general-purpose finite element code, several such codes for laminate composite structural analysis were considered in FY88. These included:

a. CODSTRAN — COmposite Durability STRuctural ANalysis (reference 13),

b. ABAQUS (reference 14),

c. MSC/NASTRAN — MacNeal-Schwendler Corporation/NAsa STRuctural ANalysis (reference 15),

d. COSMIC/NASTRAN — COmputer Software Management and Information Center/NAsa STRuctural ANalysis (reference 16).

Among these codes, COSMIC/NASTRAN was found to be the only code that encompasses all of the following features:

a. Availability of the source codes for programming implementation,

b. Maturity of the code developments,

c. Frequent use in our day-to-day work at Naval Ocean System Center.

Thus, COSMIC/NASTRAN was chosen for incorporating the proposed progressive failure model. This code has been leased on an annual basis and is currently operational at this center.

In addition to acquiring COSMIC/NASTRAN, the special-purpose finite element program, PDHOLE, developed by Stanford University (reference 7), has been purchased. This program was written specially for predicting the tensile strength of a composite laminate strip containing an open hole. Due to this special-purpose nature, this program is much simpler (shorter) than COSMIC/NASTRAN to follow. Furthermore, PDHOLE is already equipped with the progressive failure modeling scheme, except with different ply failure criterion and stiffness degradation model from what we proposed. In light of this, the improved multiaxial failure criterion as well as the associated stiffness degradation model[1] can be implemented into the PDHOLE code to gain failure model implementation experiences. Moreover, the implemented version of the PDHOLE code can be used for the subsequent program comparison and check-out purposes with the implemented version of COSMIC/NASTRAN.

Thus, the required works for implementing the two codes were planned for FY89 and FY90 in the following two subsections.

[1]The associated stiffness degradation model will be presented in subsection 4.2.

3.1 WORK PLANNED FOR FY89

Work planned for FY89 consists of

a. Code-Supporting Literature Studies — Study in detail all[1] the literature directly relevant to both the understanding of the two computer codes and the programming of the proposed progressive failure model into the two codes.

b. PDHOLE Modifications — Understand the program listings and program the improved multiaxial failure criterion and the associated stiffness degradation model into the computer program.

3.2 WORK PLANNED FOR FY90

Work planned for FY90 consists of

a. COSMIC/NASTRAN Modifications — Run COSMIC/NASTRAN with test problems; study the program listings for implementation purposes; implement a progressive failure modeling driver into the program; write subroutines for the improved failure criterion and the proposed stiffness degradation model; and program damage propagation output subroutines.

b. Code Verifications — Numerically verify the modified COSMIC/ NASTRAN, using the modified PDHOLE.

[1]Except COSMIC/NASTRAN program manuals.

4.0 RESULTS OBTAINED IN FY89

The work scheduled for FY89 as described in subsection 3.1 has been all completed. The results are highlighted below in subsections 4.1 and 4.2.

4.1 LITERATURE STUDY

To understand the PDHOLE code, a series of papers (references 6, 7, and 17 to 25) contributing to the code development during 1982 and 1987 have been studied in great depth. They provide insights into how the following basic concepts have been implemented into the code:

a. Progressive failure modeling driver,

b. Ply failure criterion,

c. Stiffness degradation model,

d. Nonlinear shear stress-strain law of lamina,

e. Stress redistribution, etc.

Due to the lack of good supporting documents of COSMIC/NASTRAN and the close programming relationship between COSMIC/NASTRAN and MSC/NASTRAN,[1] understanding of COSMIC/NASTRAN began by studying the book *MSC/NASTRAN Primer* (reference 26). This book provided the reader not only with a description of the technological content of the code, but also with a description of the NASTRAN vocabulary and capability for structural analysis. In addition to the study of this book, a short course, entitled "DMAP and Database Application in MSC/NASTRAN Version 66," was taken. This course showed the student how to perform tasks in NASTRAN. Such tasks included the creation, storage, and maintenance of user-written solution sequences, writing structured DMAP, and operation of complex database using the File Management System.

4.2 PDHOLE CODE MODIFICATION

This subsection records the results for in-plane ply failure criterion and the associated stiffness degradation model that have been implemented in the PDHOLE code. The detailed derivations of these results from the general three-dimensional theories are omitted here, but they shall be given in a forthcoming final report on the progressive failure model development. Moreover, this subsection also summarizes the programming implementation of the original version of the PDHOLE code.

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[1]As is well known, COSMIC/NASTRAN and MSC/NASTRAN have common origins in Level 15.5 NASTRAN (reference 26).

4.2.1 Ply Failure Criterion

In PDHOLE, consideration is restricted to the plane-stress state:[1]

$$\sigma_1 \neq 0, \sigma_2 \neq 0, \sigma_{12} \neq 0, \quad \sigma_3 = \sigma_{13} = \sigma_{23} = 0, \tag{8}$$

or

$$\underset{\sim}{\sigma} = \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{Bmatrix}. \tag{9}$$

Also, the composite lamina under consideration has a material symmetry no more general than orthotropic. For this material in the plane-stress state, the stress-strain relations are given by

$$\underset{\sim}{\sigma} = \underset{\sim}{Q}\, \underset{\sim}{\epsilon}, \tag{10}$$

where $\underset{\sim}{\epsilon}$ is the (ply) strain vector having three components:

$$\underset{\sim}{\epsilon} = \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_{12} \end{Bmatrix}, \tag{11}$$

and $\underset{\sim}{Q}$ is the (ply) reduced stiffness matrix given by

$$\underset{\sim}{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{44} \end{bmatrix}. \tag{12}$$

For an orthotropic composite lamina in the plane-stress state, the general multiaxial failure criterion represented by equation (3) reduces to[2]

$$\begin{aligned} f = {} & F_1 \sigma_1 + F_2 \sigma_2 + F_{11} \sigma_1^2 + 2F_{12} \sigma_1 \sigma_2 + F_{22} \sigma_2^2 + F_{44} \sigma_{12}^2 \\ & + (H_1 \sigma_1 + H_2 \sigma_2) \mid H_1 \sigma_1 + H_2 \sigma_2 \mid = 1, \end{aligned} \tag{13}$$

and there are eight independent strength constants (F_1, F_2; F_{11}, F_{12}, F_{22}, F_{44}; H_1, H_2) appearing in the ply failure criterion. The number of independent strength constants

[1]Throughout the remainder of this report, the 1-2 plane is taken to be tangent to the lamina surface and the 3-axis is taken to be along the thickness direction of the lamina (see figure 1(a)).

[2]In view of the footnote on page 4, it can be seen that $\sigma_{12} = \sigma_4$.

can be reduced to four for either a transversely isotropic or an isotropic composite lamina in the plane-stress state. For either material, the independent strength constants are: F_1, F_{11}, F_{12}; H_1, and the following relations hold:

$$F_2 = F_1 \, , \; F_{22} = F_{11} \, , \; F_{44} = 2 \, (F_{11} - F_{12}), \; H_2 = H_1 \, . \tag{14}$$

4.2.2 Stiffness Degradation Model

Once failure occurs, the material may undergo some degree of property loss in the damaged area. At the failed material point, this is realized by reducing the longitudinal modulus, E_1, the tranverse modulus, E_2, the in-plane shear modulus, G_{12}, and the major Poisson's ratio, ν_{12}, to near zero. Consequently, at the failure point, the components of the ply reduced stiffness matrix Q can be set as

$$Q_{11} = Q_{12} = Q_{22} = Q_{44} = 10^{-5} \, E_1 \, , \tag{15}$$

and all in-plane stress components are vanishing:

$$\sigma_1 = \sigma_{12} = \sigma_2 = 0 \, . \tag{16}$$

4.2.3 Programming Implementation

For programming the proposed ply failure criterion and stiffness degradation model into the PDHOLE code, a COMMON block named MP1 has been added to the appropriate subroutines in the code and various subroutines have been modified.

The MP1 COMMON block is given by:

$$COMMON/MP1/IFAIL \, , \; F1 \, , \; F2 \, , \; F11 \, , \; F12 \, , \; F22 \, , \; F44 \, , \; H1 \, , \; H2 \, , \tag{17}$$

where

IFAIL = Failure Criterion Option

= 0: Use the failure criterion originally programmed in the code,

1: Use the proposed piecewise quadratic strength tensor failure criterion.

$F1$, $F2$, $F11$, $F12$, $F22$, $F44$, $H1$, $H2$

= FORTRAN variable names for the strength constants input: F_1 , F_2 , F_{11} , F_{12} , F_{22} , F_{44} , H_1 , H_2 .

Among those which have been extensively modified are the following subroutines:

MAIN	–	Main program,
INPUT	–	Input material properties,
CFAIL	–	Ply failure criterion,
PMATRL	–	Stiffness degradation model,
STRESS	–	Stress calculation,
RDSTR	–	Stress redistribution.

REFERENCES

1. Nuismer, R. J., and G. E. Brown. 1976. "Progressive Failure of Notched Composite Laminates Using Finite Elements," *Proceedings of the 13th Annual Meeting of the Society of Eng. Sciences*, NASA CP-2001, NASA, Washington, D.C., pp. 183-192.

2. Chou, S. C., O. Orringer, and J. H. Rainey. 1977. "Post-Failure Behavior of Laminates: II-Stress Concentration," *Journal of Composite Materials*, Vol. 11, pp. 71-78.

3. Chamis, C. C., and G. T. Smith. 1980. "CODSTRAN: Composite Durability Structural Analysis," *Fibrous Composites in Structural Design*, Edited by E. M. Lenoe, D. W. Oplinger, and J. J. Burke, Plenum Press, New York and London, pp. 271-280.

4. Lee, J. D. 1982. "Three Dimensional Finite Element Analysis of Damage Accumulation in Composite Laminate," *Computers and Structures*, Vol. 15, No. 3, pp. 335-350.

5. Sanhu, R. S., G. P. Sendeckyj, and R. L. Gallo. 1982. "Modeling of the Failure Process in Notched Laminates," *Mechanics of Composite Materials — Recent Advances*, Proceedings of the IUTAM Symposium on Mechanics of Composite Materials, Virginia Polytechnic Institute and State University, Balcksburg, Virginia, USA; Edited by Z. Hashin and C. T. Herakovich, pp. 179-189.

6. Chang, F. K. 1986. "A Progressive Damage Model for Laminated Composites Containing an Open or a Pin-Loaded Hole," 18th International SAMPE Technical Conference, pp. 360-371.

7. Chang, F. K. 1987. "A Progressive Damage Model for Laminated Composites Containing Stress Concentrations," *Journal of Composite Materials*, Vol. 21, pp. 834-855.

8. Tang, P. Y. 1987. *A Piecewise Quadratic Strength Tensor Theory for Composites*, Naval Ocean Systems Center, TR 1188.

9. Tang, P. Y. 1988. "A Multiaxial Failure Criterion for Composites," *Computational Probabilistic Methods*, AMD Vol. 93, Edited by W. K. Liu, et al., pp. 87-96.

10. Jones, R. M. 1975. *Mechanics of Composite Materials*, McGraw-Hill Book Company.

11. Tsai, S. W., and H. T. Hahn. 1980. *Introduction to Composite Materials*, Technomic Publishing Company, Inc.

12. Vinson, J. R., and R. L. Sierakowski. 1986. *The Behavior of Structures Composed of Composite Materials*, Martinus Nijhoff Publishers.

13. Chamis, C. C., and G. T. Smith. 1978. *CODSTRAN: Composite Durability Structural Analysis*, NASA TM-79070.

14. *ABAQUS Manuals*, Version 4.7, 1988. Hibbitt, Karlsson & Sorensen, Inc.

15. *MSC/NASTRAN Manuals*, Version 64, 1986. MacNeal-Schwendler Corp.

16. *COSMIC/NASTRAN Manuals*, NASA SP-221(06), 1981; NASA SP-222(08), 1986; NASA SP-223(05), 1978; NASA SP-224(05), 1978.

17. Chang, F. K., R. A. Scott, and G. S. Springer. 1982. "Strength of Mechanically Fastened Composite Joints," *Journal of Composite Materials*, Vol. 16, pp. 470-493.

18. Chang, F. K., R. A. Scott, and G. S. Springer. 1984. "Failure of Composite Laminates Containing Pin Loaded Holes — Method of Solution," *Journal of Composite Materials*, Vol. 18, pp. 225-278.

19. Chang, F. K., R. A. Scott, and G. S. Springer. 1984. "Failure Strength of Nonlinearly Elastic Composite Laminates Containing a Pin Loaded Hole," *Journal of Composite Materials*, Vol. 18, pp. 464-477.

20. Chang, F. K. 1986. "The Effect of Pin Load Distribution on the Strength of Pin Loaded Holes in Laminated Composites," *Journal of Composite Materials*, Vol. 20, pp. 401-408.

21. Chang, F. K., and W. H. Chen. 1987. "In Situ Shear Strength Distribution in Laminated Composites," *Journal of Composite Materials*, Vol. 21, pp. 708-733.

22. Chang, F. K., and K. Y. Chang. 1987. "Post-Failure Analysis of Bolted Composite Joints in Tension and Shear-out Mode Failure," *Journal of Composite Materials*, Vol. 21, pp. 809-833.

23. Zienkiewicz, O. C. 1977. *The Finite Element Method*, McGraw-Hill, New York.

24. Hahn, H. T., and S. W. Tsai. 1973. "Nonlinear Elastic Behavior of Unidirectional Composite Laminate," *Journal of Composite Materials*, Vol. 7, pp. 102-110.

25. Yamada, S. E., and C. T. Sun. 1978. "Analysis of Laminate Strength and its Distribution," *Journal of Composite Materials*, Vol. 12, pp. 275-284.

26. Schaeffer, H. G. 1984. *MSC/NASTRAN Primer: Static and Normal Modes Analysis*, Schaeffer Analysis, Inc., Mont Vernon, New Hampshire.